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ON ANGULAR MOMENTUM TRANSFER IN BINARY SYSTEMS

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SUMMARY

The limiting efficiency is considered with which orbital angular momentum can be converted into rotational angular momentum, J_{rot}, of the mass-gaining component in binary systems which undergo mass exchange. This limit, $(dJ/dM)_{max}$, then specifies the maximum extent to which the observed rates of period change, dP/dt, can be affected by such reduction of orbital angular momentum. In most cases this process cannot seriously affect dP/dt, and therefore can be safely neglected in computing mass transfer rates and in comparing observed dP/dt values with evolutionary model calculations. Upon integrating $(dJ/dM)_{max}$ over the entire accretion process, we find that the maximum accumulated rotational angular momentum is larger than the amount implied by the observed underluminosities of stars in certain extreme binary systems, by factors of 3 to 4. Shell stars and emission-line stars in binary systems may be produced when core angular momentum is later transferred into an envelope which already has nearly the limiting J_{rot}.

1. INTRODUCTION

Observed rates of orbital period change (dP/dt) are sometimes used to calculate mass transfer rates in close binary systems. They are also used in the matching of theoretical evolutionary model calculations, which predict dP/dt as a function of time, to observed systems. Usually, conservation of the total mass of the system and of total orbital angular momentum are assumed in such computations. For both purposes one could object that an observed value of dP/dt might be affected to a serious extent by conversion of orbital angular momentum into rotational angular momentum of the mass-gaining component. In this paper we discuss quantitatively the maximum orbital period change and the maximum angular momentum conversion that can be expected.

2. PERIOD CHANGES

The relation between the orbital period, P, and the orbital angular momentum, J, which follows from Kepler's third law,

$$P = \frac{2\pi(M_1+M_2)}{G^2}\frac{J^3}{(M_1M_2)^3}, \tag{1}$$

leads, upon being differentiated, to

$$\frac{dP}{dt} = \frac{6\pi(M_1+M_2)\,J^2}{G^2(M_1M_2)^3}\left[\frac{dJ}{dt} - \frac{J(M_1-M_2)}{M_1M_2}\frac{dM_2}{dt}\right]. \tag{2}$$

M_1 and M_2 refer to the mass-losing and mass-gaining components, respectively.

Thus the two terms in parentheses show the relative importance of the non-conservation of J and the mass transfer in establishing dP/dt. Of course, orbital angular momentum which goes into rotational angular momentum behaves, in so far as period changes are concerned, just as if it had been lost to the system, so we shall refer to this effect as non-conservation of J(NCJ). We wish to consider under what circumstances NCJ can be neglected. We therefore consider the ratio of these terms,

$$\frac{dP_{\mathrm{NCJ}}}{dP_{\mathrm{MT}}} = -\frac{M_1 M_2}{J(M_1 - M_2)}\frac{dJ}{dM_2}, \tag{3}$$

where MT denotes *mass transfer*. We can place a limit on dJ/dM_2 as follows. An element, m, of accreting mass will transfer the most angular momentum when it strikes the accreting star tangentially with the 'velocity from infinity', which is $\sqrt{2}$ times larger than the Keplerian circular velocity for a given radius. One can then readily show that the angular momentum of m will be given by

$$J_{\mathrm{m}} = km(2GM_2R_2)^{1/2} \tag{4}$$

where k is a factor which depends on the latitude at which the gas stream, assumed to be moving parallel to the orbital plane, hits the surface, and which therefore lies between zero and unity. For most realistic cases, $\langle k \rangle$ will be between 1/2 and unity. Clearly,

$$dJ/dM_2 = -k(2GM_2R_2)^{1/2}. \tag{5}$$

Putting (5) into (3) and substituting for J from (1) yields

$$\frac{dP_{\mathrm{NCJ}}}{dP_{\mathrm{MT}}} = \left(\frac{32\pi^2}{G}\right)^{1/6} k \left[\frac{(1+q)^{1/3}q^{1/2}}{(1-q)} \frac{R_2^{1/2}}{M_1^{1/6}P^{1/3}}\right] \tag{6}$$

where $q = M_2/M_1$. Let n be the exponent in the mass–radius relation for the accreting star, so that

$$R_2 = R_{\odot}(M_2/M_{\odot})^n. \tag{7}$$

Putting (7) into (6), we find

$$\frac{dP_{\mathrm{NCJ}}}{dP_{\mathrm{MT}}} = \left(\frac{32\pi^2 R_{\odot}^3}{GM_{\odot}}\right)^{1/6} k \left[\frac{(1+q)^{1/3}q^{(n+1)/2}}{(1-q)} \frac{(M_1/M_{\odot})^{(3n-1)/6}}{P^{1/3}}\right]. \tag{8}$$

We now adopt $n = 0{\cdot}55$ according to the normal mass–radius relation for non-rotating ZAMS stars. Although an accreting component certainly will not be a simple ZAMS star, since it will be disturbed by rotational and tidal distortion, circulation currents, and surface impacts, nevertheless its chemical composition should remain well mixed and therefore should be changed very little by the final accretion of some helium-rich material from the core of the other star (Stothers 1972). Although R_2 may be abnormally large during the most rapid phase of mass transfer (Benson 1970), the general form of equation (8) permits the use of other values of n, should this seem to be required. Furthermore the strictly rotational and tidal effects on the radius are expected to be relatively small and can be neglected here. A larger value of n would not significantly alter our general conclusions.

Numerically, with P in days, we find

$$\frac{dP_{\mathrm{NCJ}}}{dP_{\mathrm{MT}}} = 0{\cdot}69\, k \left[\frac{(1+q)^{1/3}q^{0{\cdot}77}}{(1-q)} \frac{(M_1/M_{\odot})^{0{\cdot}11}}{P^{1/3}}\right]. \tag{9}$$

As an example, consider the case of β Lyrae, for which we adopt the parameters $M_1 = 2\,M_\odot$, $P = 12{\cdot}9$, $k = 0{\cdot}8$, and $q = 6$ (Wilson 1974). Equation (9) then gives

$$(dP/dt)_{\mathrm{NCJ,\,max}} = -0{\cdot}39(dP/dt)_{\mathrm{MT}}. \tag{10}$$

This turns out not to be so large as to affect the sign of the period change or even its magnitude to a large extent. Therefore, for these parameter values, NCJ can safely be neglected. Nevertheless, note that, for $q > 1$, neglect of NCJ will yield a mass transfer rate which is a *lower limit* to the true rate. Equation (9) shows a very weak dependence on M_1, and the principal dependence is on the mass ratio Indeed, even with $q = 2$ (and other parameters above unchanged) we find that $dP_{\mathrm{NCJ}}/dP_{\mathrm{MT}}$ is only $-0{\cdot}64$, so that the effect of NCJ is essentially negligible, since it cannot change the order of magnitude of dP/dt. Only when q approaches unity does the dJ/dt term become dominant, and this is only because the orbital term vanishes when the masses are equal.

One might also consider tidal transfer of rotational angular momentum back into the orbit, as Biermann & Hall (1973) and Stothers (1973) have recently done in somewhat different contexts. Since this effect gives a $(dP/dt)_{\mathrm{TT}}$ term (TT = tidal transfer) of the same sign (if $q > 1$) as $(dP/dt)_{\mathrm{MT}}$, the absolute value of $(dP/dt)_{\mathrm{TT}}$ would need to be as large as $(dP/dt)_{\mathrm{MT}}$ in order to introduce as much as a factor of two error. Therefore this effect probably can also be safely neglected in most cases. However, we have not attempted to estimate $(dJ/dt)_{\mathrm{TT,\,max}}$.

We consider now a second way to specify R_2. Suppose the transferred material goes into orbit around the secondary star instead of being accreted directly. If so, the appropriate radius for use in equation (6) is the ring or disk radius at which the material achieves circular orbit. We then find, substituting for P in equation (6) by means of Kepler's third law,

$$\frac{dP_{\mathrm{NCJ}}}{dP_{\mathrm{MT}}} = k(2R_2/a)^{1/2}\left[\frac{(1+q)^{1/2}q^{1/2}}{(1-q)}\right]. \tag{11}$$

R_2/a cannot be as large as the secondary Roche lobe, because the Keplerian circular velocity is larger than the binary orbital velocity at that distance from M_2 by a factor of about 2. For simplicity, however, we shall use the Roche lobe radius for R_2/a and thus find a generous upper limit for $dP_{\mathrm{NCJ}}/dP_{\mathrm{MT}}$. For $q = 6$ the smallest equatorial radius (R_2/a) of the secondary lobe is about $0{\cdot}55$ (Kopal 1959), and equation (11) yields $dP_{\mathrm{NCJ}}/dP_{\mathrm{MT}} \approx -1{\cdot}0$. This corresponds to complete cancellation of normal period changes by the NCJ effect. However, we know from our (unpublished) orbit integrations and from those of many other authors (e.g. Kruszewski 1964) that material which falls from the inner Lagrangian point into the lobe of an accreting star deviates rather little from the line of centres in the rotating coordinate frame, and will strike the secondary star directly unless the relative radius of that star is rather small. Thus we can see in this simple way that the *available* angular momentum carried by the material is small enough so that it is our former limit, based on the secondary star radius, and not that based on the Roche lobe radius, which applies. In fact, this suggests that it should be possible to set a more restrictive limit on $dP_{\mathrm{NCJ}}/dP_{\mathrm{MT}}$ by considering the limited angular momentum carried by the transferred mass.

3. ROTATIONAL CHANGES

The foregoing considerations also lead to a limit on the total accreted angular momentum. Integrating (5) over the entire accretion process from an initial stage i to a final stage f, we find

$$\Delta J = \frac{2k}{n+3}(2GR_\odot M_\odot{}^3)^{1/2}\left[\left(\frac{M_{2,f}}{M_\odot}\right)^{(n+3)/2} - \left(\frac{M_{2,i}}{M_\odot}\right)^{(n+3)/2}\right]. \qquad (12)$$

Stothers & Lucy (1972) showed that in certain massive binary systems with interacting components the accreting star is underluminous with respect to a normal uniformly rotating ZAMS star. They attributed the underluminosity to high rotational angular momentum in the core acquired during the mass transfer process. Since the amount of underluminosity depends theoretically only on the total rotational angular momentum, J_{rot}, and not on its distribution within the star (Bodenheimer 1971), it proved possible to determine J_{rot} from the observed underluminosity (Stothers 1973). We can now check these values against the upper

TABLE I

'Observed' J_{rot} and maximum ΔJ_{rot} for underluminous binary components

System	M_1+M_2 ($M_\odot$)	$J_{\mathrm{orb}}\times 10^{-53}$ (g cm^2 s^{-1})	$J_{\mathrm{rot}}/J_{\mathrm{orb}}$ (observed)	$\Delta J_{\mathrm{rot}}/J_{\mathrm{orb}}$ (maximum)
μ^1 Sco	9+14	6·2	0·19	0·67
V Pup	10+18	8·3	0·25	0·78
SX Aur	6+11	3·4	0·22	0·82
V356 Sgr	5+12	5·7	0·18	0·56

limits given by equation (12). The results are shown in Table I for four systems in which the 'observed' J_{rot} averages 0·21 times the orbital angular momentum, J_{orb}. Since these systems are clearly in the slow phase of mass transfer, we can adopt $n = 0{\cdot}55$ with little uncertainty. We also adopt $k = 0{\cdot}8$, as before. Equation (12) then gives

$$\Delta J_{\mathrm{rot}} = 3{\cdot}8\times 10^{51}\left[\left(\frac{M_{2,f}}{M_\odot}\right)^{1\cdot 775} - \left(\frac{M_{2,i}}{M_\odot}\right)^{1\cdot 775}\right] \text{ g cm}^2\text{ s}^{-1}. \qquad (13)$$

Evaluation of (13) for $M_{2,i} \ll M_{2,f}$, which is an adequate approximation for our present purpose, gives the last column in Table I, $\Delta J_{\mathrm{rot}}/J_{\mathrm{orb}}$. Clearly the 'observed' values of $J_{\mathrm{rot}}/J_{\mathrm{orb}}$ fall well below the theoretical limiting values, by factors of 3 to 4. Nevertheless, not all of the apparent underluminosity need be due to fast differential rotation if these stars possess an occluding disk, as in β Lyrae (Wilson 1974).

If these stars did at one time have the theoretical maximum values of J_{rot}, say during the rapid phase of mass transfer, plausible reasons might explain why they do not at present. For example, tides due to the companion will act to retard rotation and to transfer rotational angular momentum back into the orbit. More importantly in wider systems where the tides are relatively weak, rotational currents alone will try to straighten out the angular velocity distribution (Zahn 1973); therefore, in time, the envelope will spin faster at the expense of the core (which could earlier have accumulated and stored a large amount of angular momentum) until mass and angular momentum are lost at the surface. The primary may then appear as an emission-line or shell star. Undoubtedly other mechanisms can produce such stars, e.g. by the earlier accretion process or by equatorially unstable rotation in even a single star, and we do not advance our mechanism as the only possible one for explaining these stars.

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